Navlakhi®

Mechanics

Chapter 2: Equilibrium

Solutions

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By Kunal Navlakhi
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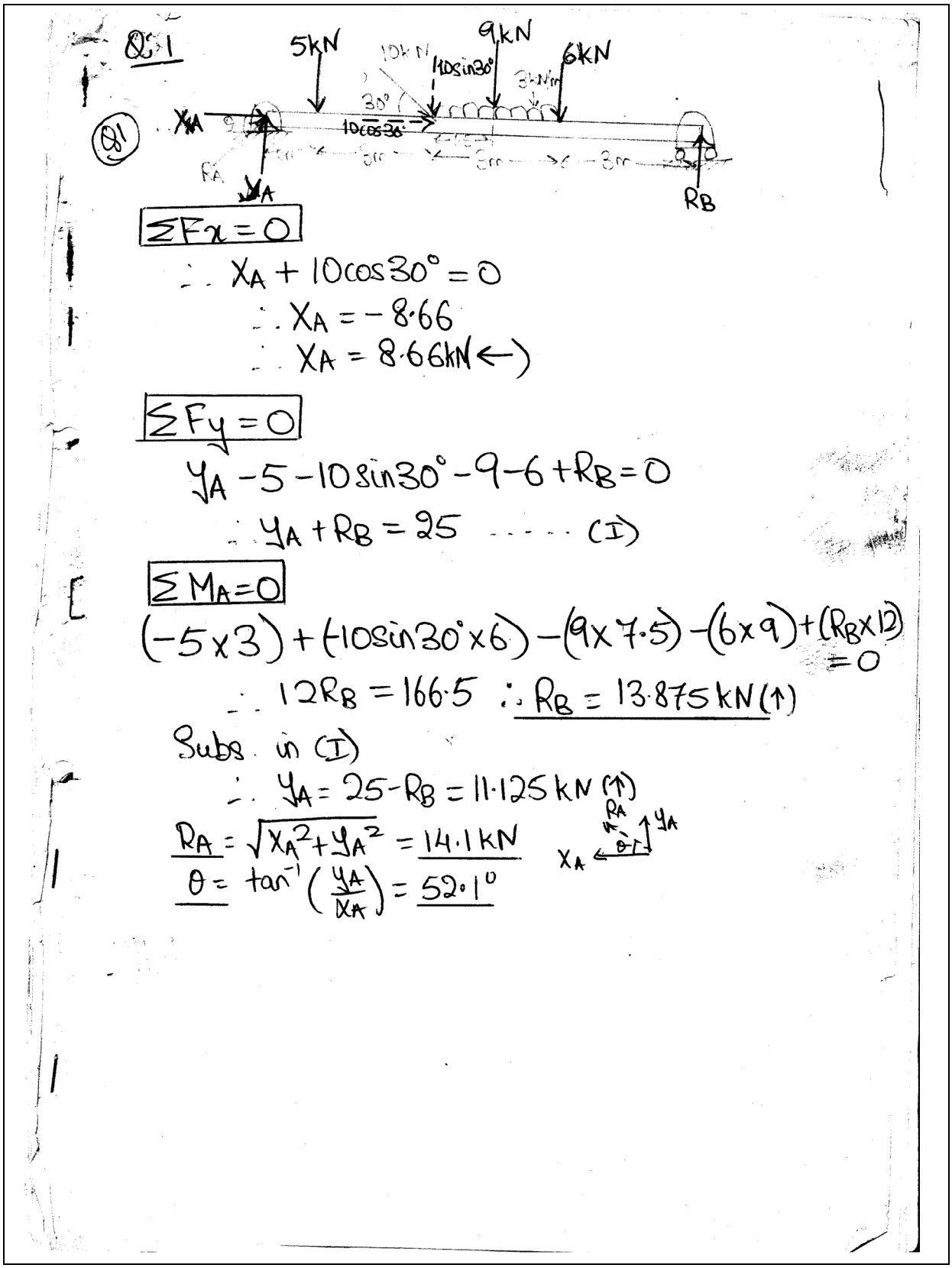


Q.1

$\Sigma F_x = 0$

$\therefore X_A + 10\cos 30° = 0$

$\therefore X_A = -8.66$

$\therefore X_A = 8.66\,kN (\leftarrow)$

$\Sigma F_y = 0$

$Y_A - 5 - 10\sin 30° - 9 - 6 + R_B = 0$

$\therefore Y_A + R_B = 25 \quad \ldots\ldots (I)$

$\Sigma M_A = 0$

$(-5 \times 3) + (10\sin 30° \times 6) - (9 \times 7.5) - (6 \times 9) + (R_B \times 12) = 0$

$\therefore 12R_B = 166.5 \quad \therefore R_B = 13.875\,kN (\uparrow)$

Subs. in (I)

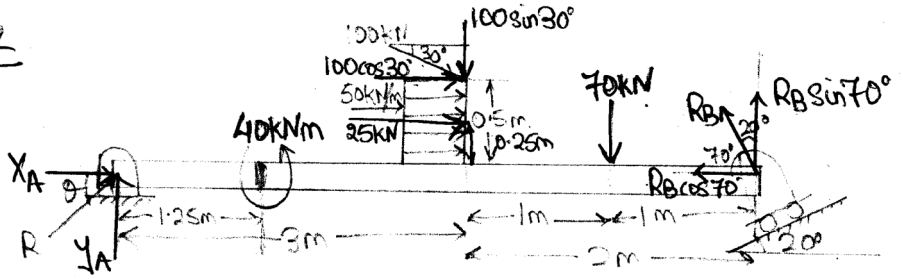
$\therefore Y_A = 25 - R_B = 11.125\,kN (\uparrow)$

$R_A = \sqrt{X_A^2 + Y_A^2} = 14.1\,kN$

$\theta = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 52.1°$

Q3.2

(Q2)



$\Sigma F_x = 0$

$$X_A + 100\cos 30° + 25 - R_B \cos 70° = 0$$

$$\therefore X_A - 0.342 R_B = -111.6 \quad \ldots (I)$$

$\Sigma F_y = 0$

$$Y_A - 100\sin 30° - 70 + R_B \sin 70° = 0$$

$$\therefore Y_A + 0.94 R_B = 120 \quad \ldots (II)$$

$\Sigma M_A = 0$

$$+40 - (25 \times 0.25) - (100\cos 30° \times 0.5) - (100\sin 30° \times 3) - (70 \times 4) + (R_B \sin 70° \times 5) = 0$$

$$4.7 R_B = 439.55$$

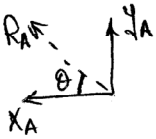
$$\therefore R_B = 93.52\,kN \text{ (↑) at } \theta = 70° \text{ in II}^{nd} \text{ quadrant}$$

Subs in (I) & (II)

$$\therefore X_A = -79.62\,kN$$

$$\therefore X_A = 79.62\,kN \text{ (←)}$$

$$Y_A = 32.1\,kN \text{ (↑)}$$

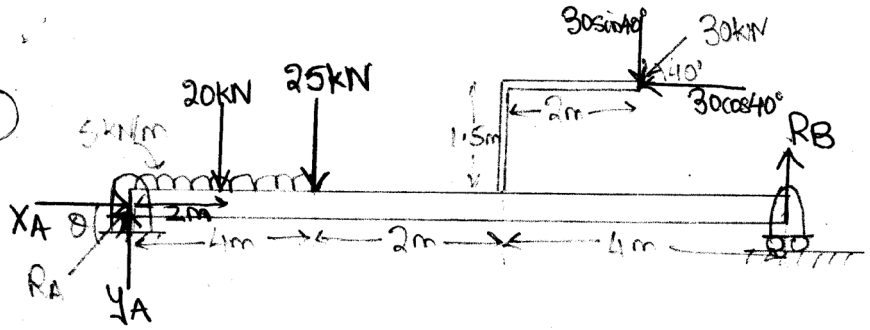


$$R_A = \sqrt{X_A^2 + Y_A^2} = 85.85\,kN$$

$$\theta = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 21.96°$$

3.3

(B3)



$\Sigma F_x = 0$

$X_A - 30\cos 40° = 0$

$X_A = 22.98\,kN\ (\rightarrow)$

$\Sigma F_y = 0$

$Y_A - 20 - 25 - 30\sin 40° + R_B = 0$

$\therefore Y_A + R_B = 64.28\,kN \quad \ldots (II)$

$\Sigma M_A = 0$

$(-20 \times 2) - (25 \times 4) - (30\sin 40° \times 8) + (R_B \times 10) + (30\cos 40° \times 1.5) = 0$

$\therefore 10R_B = 259.8\,kN$

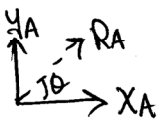
$R_B = 25.98\,kN\ (\uparrow)$

Subs. in (I)

$\therefore Y_A = 38.3\,kN\ (\uparrow)$

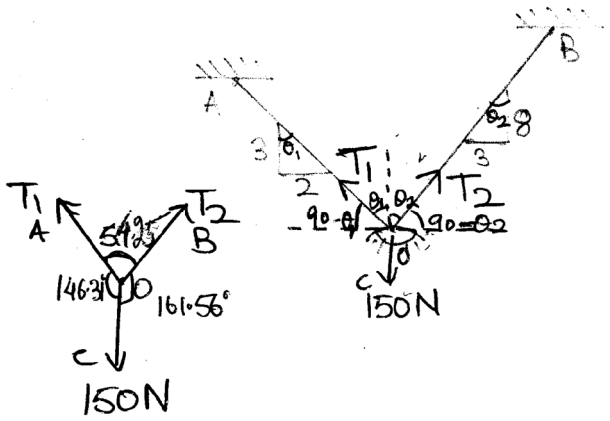
$R_A = \sqrt{X_A^2 + Y_A^2} = 44.67\,kN$

$\theta = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 59.04°$



Q3.4

(Q4)



$\theta_1 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$

$\theta_2 = \tan^{-1}\left(\frac{3}{8}\right) = 20.56°$

$\theta_1 + \theta_2 = 54.25°$

$\angle AOB = 54.25°$

$\angle AOC = (90-\theta_1)+90 = 180-\theta_1 = 146.31°$

$\angle BOC = (90-\theta_2)+90 = 180-\theta_2 = 159.44°$

By Lami's Theorem

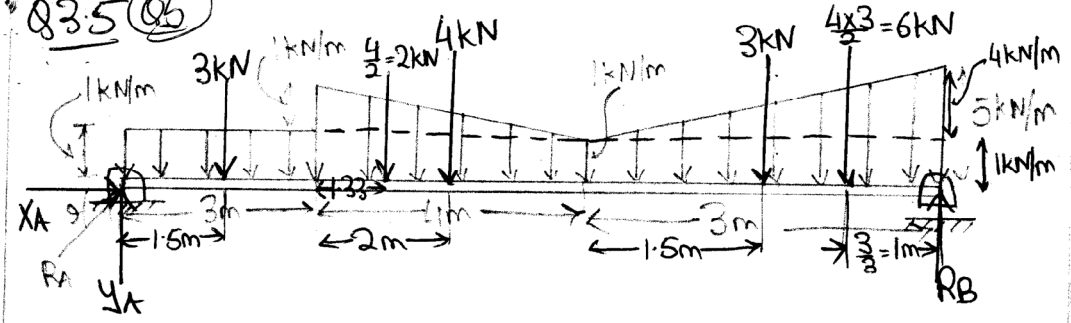
(Can be used only for three concurrent forces)

$$\frac{150}{\sin 54.25°} = \frac{T_1}{\sin 159.44} = \frac{T_2}{\sin 146.31°}$$

$\therefore T_1 = 64.9\ N$

$\therefore T_2 = 102.9\ N$

Q3.5 (05)



$\Sigma F_x = 0$

$\therefore X_A = 0$

$\Sigma F_y = 0$

$\therefore Y_A - 3 - 2 - 4 - 3 - 6 + R_B = 0$

$\therefore Y_A + R_B = 18 \quad \ldots\ldots (I)$

$\Sigma M_A = 0$

$-(3 \times 1.5) - (2 \times 4.33) - (4 \times 5) - (3 \times 8.5) - (6 \times 9) + (R_B \times 10) = 0$

$\therefore 10 R_B = 112.66$

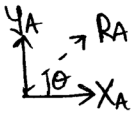
$\therefore R_B = 11.266 kN (\uparrow)$

Subs in (I)

$\therefore Y_A = 6.73 kN (\uparrow)$

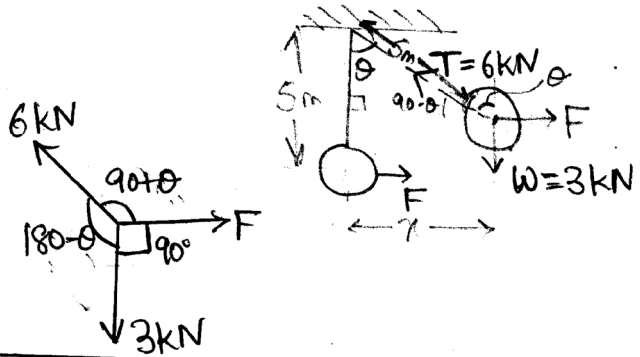
$R_A = \sqrt{X_A^2 + Y_A^2} = 6.73 kN$

$\theta = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 90°$



Q3.6

(Q6)



By Lami's Theorem

$$\frac{6}{\sin 90^\circ} = \frac{3}{\sin(90+\theta)} = \frac{F}{\sin(180-\theta)}$$

$\therefore \sin(90+\theta) = 0.5$

$\therefore 90+\theta = 30^\circ \text{ or } 150^\circ$

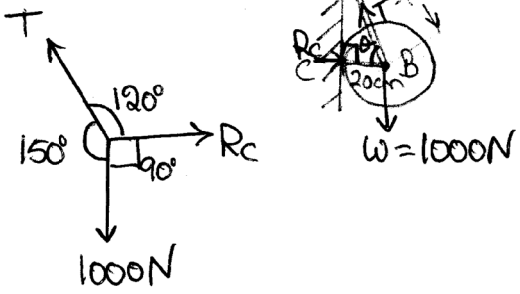
$\therefore \theta = 150-90^\circ = 60^\circ$

$\sin\theta = \frac{x}{5} \quad \therefore x = 4.33\text{m}$

$F = 6 \times \sin 120^\circ = 5.196\text{ kN}$

Q3.7

(87)



$\cos\theta = \frac{20}{40} \quad \therefore \theta = 60°$

By Lami's Theorem.

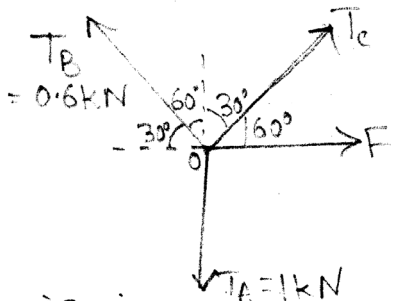
$$\frac{1000}{\sin 120°} = \frac{T}{\sin 90°} = \frac{Rc}{\sin 150°}$$

$\therefore T = 1154.7N$

$\therefore Rc = 577.35N (\rightarrow)$

Q3.8

(88)



Since boat is in equilibrium.

$\Sigma F_x = 0$

$F + T_C \cos 60° - T_B \cos 30° = 0$

$\therefore F + 0.5T_C = 0.866 T_B$

$F + 0.5T_C = 0.52$ .... (I)

$\Sigma F_y = 0$

$T_B \sin 30° + T_C \sin 60° - T_A = 0$

$\therefore 0.3 + 0.866 T_C - 1 = 0$

$\therefore T_C = 0.81\,kN$

Sub. in (I)

$\therefore F = 0.115\,kN\ (\rightarrow)$

OC Breaks:-

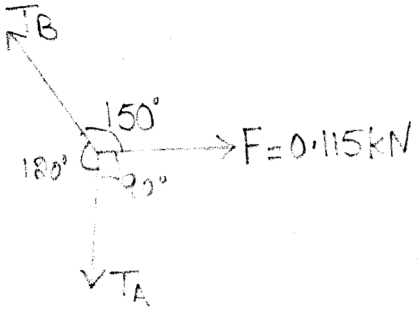
Boat remains in equilibrium

By Lami's Theorem,

$$\frac{F}{\sin 120°} = \frac{T_B}{\sin 90°} = \frac{T_A}{\sin 150°}$$

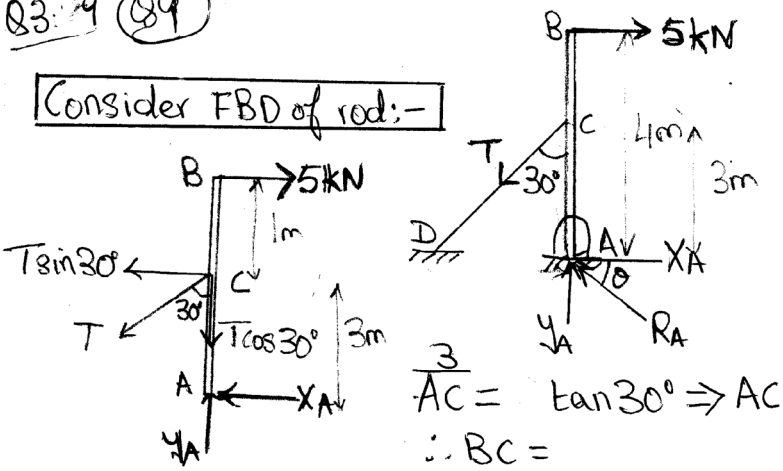
$\therefore T_B = 0.133\,kN$

$\therefore T_A = 0.066\,kN$



Q3: 9 (89)

Consider FBD of rod:-



$\frac{3}{AC} = \tan 30° \Rightarrow AC = 5.2$

$\therefore BC =$

$\Sigma F_x = 0$

$5 - T\sin 30° - X_A = 0$

$\therefore X_A + 0.5T = 5 \quad \ldots\ldots (I)$

$\Sigma F_y = 0$

$Y_A - T\cos 30° = 0$

$\therefore Y_A = 0.866T \quad \ldots\ldots (II)$

$\Sigma M_A = 0$

$-(5 \times 4) + (T\sin 30° \times 3) = 0$

$\therefore 1.5T = 20$

$\therefore T = 13.33kN$

Subs. in (I) and (II)

$\therefore Y_A = 11.54kN (\uparrow)$

$\therefore X_A = -1.67kN$

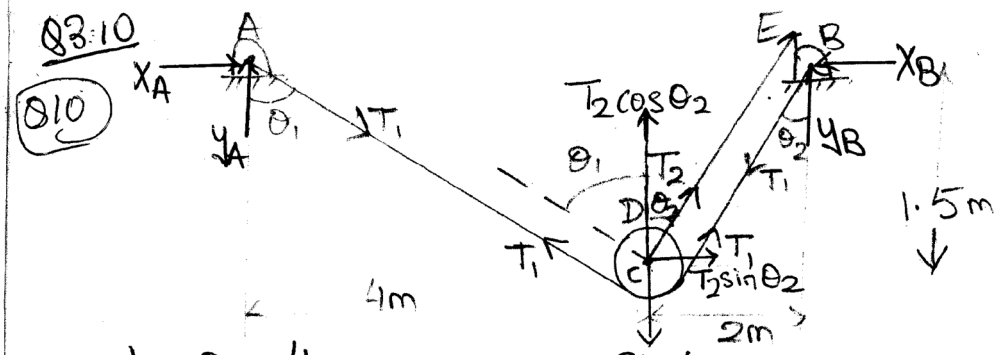
$\therefore X_A = 1.67kN (\rightarrow)$

$R_A = \sqrt{X_A^2 + Y_A^2} = 11.66\ kN$

$\theta = \tan^{-1}(Y_A / X_A) = 81.77°$

Q3:10

Q10

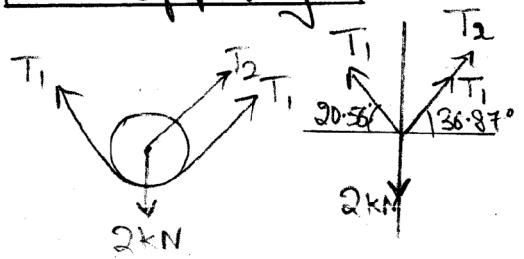


$\tan\theta_1 = \frac{4}{1.5}$

$\therefore \theta_1 = 69.44°$

$\tan\theta_2 = \frac{2}{1.5}$ $\therefore \theta_2 = 53.13°$

FBD of pulley:-



$\Sigma F_x = 0$

$T_1\cos 36.87° + T_2\cos 36.87° - T_1\cos 20.56° = 0$

$\therefore 0.8T_2 = 0.136T_1$

$\therefore T_2 = 0.17T_1$ ... (I)

$\Sigma F_y = 0$

$\therefore T_1\sin 36.87 + T_2\sin 36.87 + T_1\sin 20.56 = 2$

$0.951T_1 + 0.6T_2 = 2$ ... (II)

From (I) & (II)

$T_2 = 1.9$ kN

$T_2 = 0.323$kN

$\Sigma M_A = 0$; Considering Entire System

$\therefore -(Y_B \times 6) - (T_1\cos\theta_2 \times 4) + (T_2\sin\theta_2 \times 1.5) + (2 \times 4) = 0$

$\therefore -6Y_B = -8.4$

$\therefore Y_B = 1.4$kN

$\Sigma M_B = 0$

$-(Y_A \times 6) - (T_2 \cos\theta_2 \times 2) + (T_2 \sin\theta_2 \times 1.5) + (2 \times 2) = 0$

$\therefore Y_A = 0.667$ kN

$\Sigma F_x = 0$

$X_A + T_2 \sin\theta_2 - X_B = 0$

$\therefore X_A - X_B = -0.258$ ..... (III)

$\Sigma M_C = 0$

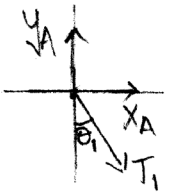
$-(X_A \times 1.5) - (Y_A \times 4) + (X_B \times 1.5) + (Y_B \times 2) = 0$

$\therefore -1.5X_A + 1.5X_B = 0.388$

$\therefore -X_A + X_B = 0.259$ .... (IV)

From (III) & (IV)

Consider FBD of A:-

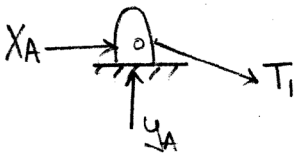


$\Sigma F_x = 0$

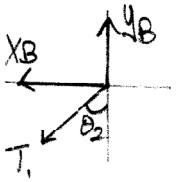
$X_A = -T_1 \sin\theta_1 = -1.78$ kN

$Y_A = T_1 \cos\theta_1 = 0.667$ kN

$\therefore \Sigma F_y = 0$ ✓



Consider FBD of B:-

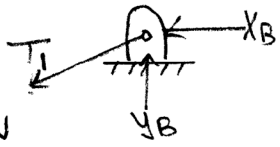


$\Sigma F_x = 0$

$X_B = -T_1 \sin\theta_2 = -1.52$ kN

$\Sigma F_y = 0$

$Y_B = T_1 \cos\theta_2 = 1.14$ kN

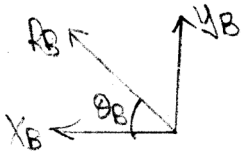


$R_A = \sqrt{X_A^2 + Y_A^2} = 1.9$ kN

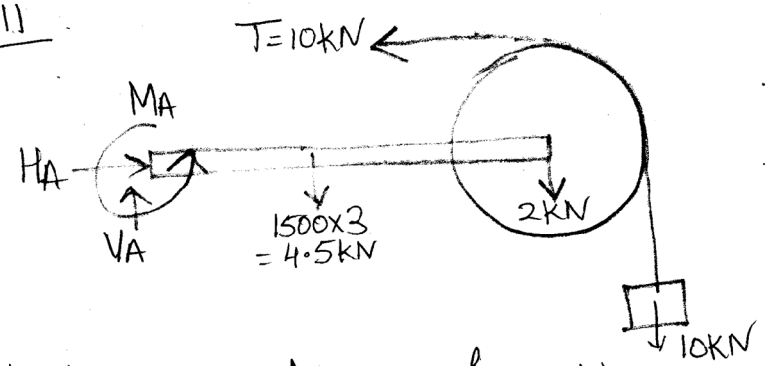
$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 20.54°$

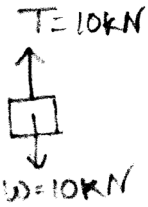
$R_B = \sqrt{X_B^2 + Y_B^2} = 1.9$ kN

$\theta_B = \tan^{-1}\left(\frac{Y_B}{X_B}\right) = 36.87°$



3.11





Applying conditions of equilibrium

$\because \Sigma F_x = 0$

$H_A - 10 = 0$

$\therefore H_A = 10\,kN\ (\rightarrow)$

$\Sigma F_y = 0$

$V_A - 4.5 - 2 - 10 = 0$

$\therefore V_A = 16.5\,kN\ (\uparrow)$

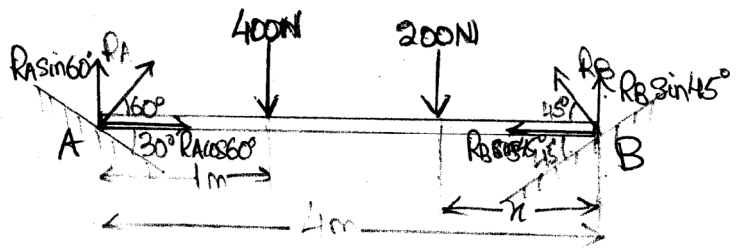
$\Sigma M_A = 0$

$4.5 \times 1.5 + 2 \times 3 + 10 \times 3.6 - T \times 0.6 - M_A = 0$

$\therefore M_A = 42.75\,kNm$

Q3.12

Q12



$\Sigma F_x = 0$

$R_A \cos 60° - R_B \cos 45° = 0$

$\therefore R_B = 0.707 R_A$

$\Sigma F_y = 0$

$R_A \sin 60° - 400 - 200 + R_B \sin 45° = 0$

$\therefore R_A = 439.25\ N$

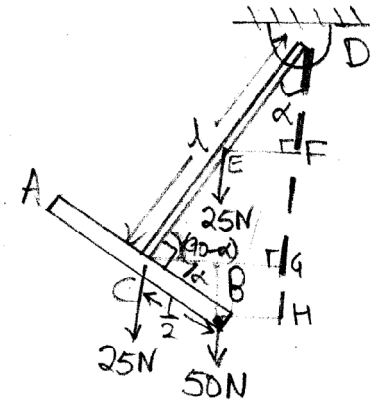
$\Sigma M_B = 0$

$-(R_A \sin 60° \times 4) + (400 \times 3) + (200 \times x) = 0$

$\therefore x = 1.607\ m$

Q3.13

Q13



$EF = \frac{l}{2} \cdot \sin\alpha$

$CG = l \cdot \sin\alpha$

$BC = \frac{l}{2} \cdot \sin\alpha\cos\alpha$

$\therefore BH = CG - BC$

$= l\sin\alpha - \frac{l}{2}\cos\alpha$

$\boxed{\Sigma M_D = 0}$

$\therefore (25 \times EF) + (25 \times CG) + (50 \times BH) = 0$

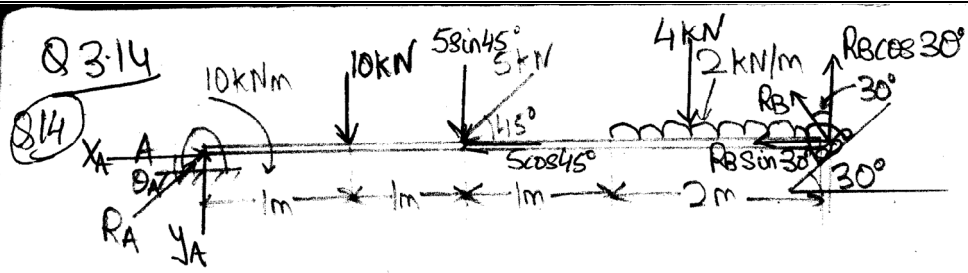
$\therefore 25 \times \frac{l}{2}\sin\alpha + 25 \times l \times \sin\alpha + 50 \times l \sin\alpha - 50 \times \frac{l}{2} \cdot \cos\alpha = 0$

$\therefore 87.5\sin\alpha = \frac{50}{2}\cos\alpha$

$\therefore \tan\alpha = 0.286$

$\therefore \alpha = 15.95^\circ$

Student can solve by taking B to the Right of Point D



$\Sigma M_A = 0$

$-(10\times1)-(5\sin45°\times2)-(4\times4)+(R_B\cos30°\times5)-10=0$

$\therefore R_B = 9.947\ kN$ at $\theta = 60°$ in IInd quadrant

$\Sigma F_x = 0$

$X_A - 5\cos45° - R_B\sin30° = 0$

$\therefore X_A = 8.51\ kN\ (\rightarrow)$

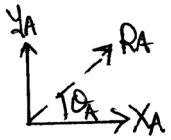
$\Sigma F_y = 0$

$Y_A - 10 - 5\sin45° - 4 + R_B\cos30° = 0$

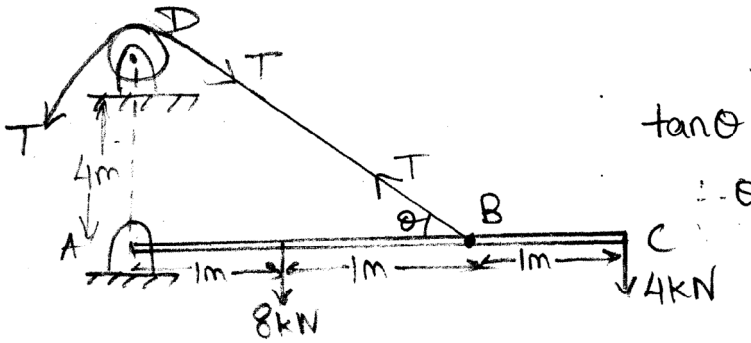
$\therefore Y_A = 8.92\ kN\ (\uparrow)$

$R_A = \sqrt{X_A^2 + Y_A^2} = 12.33\ kN$

$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 46.35°$



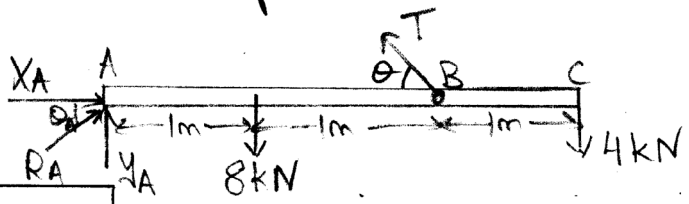
Q3.15



$\tan\theta = \frac{4}{2}$

$\therefore \theta = 63.44°$

Consider FBD of AC:-



$\Sigma M_A = 0$

$\therefore -(8\times1) + (T\sin\theta \times 2) - (4\times3) = 0$

$\therefore T = 11.18\ kN$

$\Sigma F_x = 0$

$X_A - T\cos\theta = 0$

$\therefore X_A = 5kN\ (\rightarrow)$

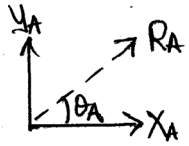
$\Sigma F_y = 0$

$Y_A - 8 + T\sin\theta - 4 = 0$

$\therefore Y_A = 2kN\ (\uparrow)$

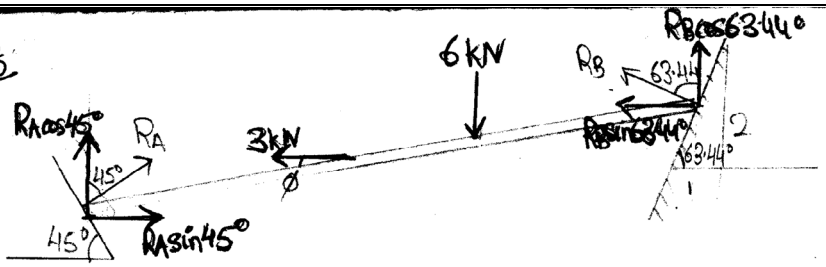
$R_A = \sqrt{X_A^2 + Y_A^2} = 5.385\ kN$

$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 21.8°$

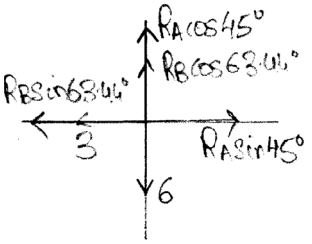


Q3.16

(16)



To obtain R_A & R_B



$\Sigma F_x = 0$

$R_A \sin 45° - 3 - R_B \sin 63.44° = 0$

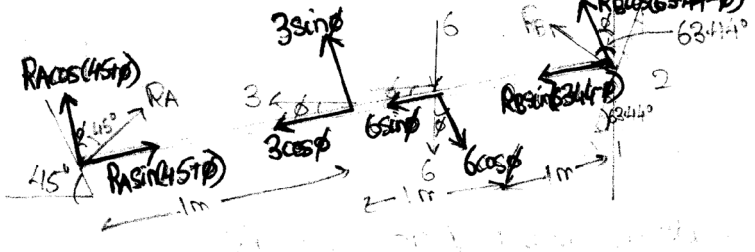
$\therefore R_A = 4.24 + R_B \times 1.265$ — (I)

$\Sigma F_y = 0$

$R_A \cos 45° + R_B \cos 63.44° - 6 = 0$ — (II)

From (I) & (II)

$R_B = 2.24$ kN, $R_A = 7.08$ kN



$\Sigma F_x = 0$

$R_A \sin(45+\phi) - 3\cos\phi - 6\sin\phi - R_B \sin(63.44-\phi) = 0$

$R_A \sin 45° \cos\phi + R_A \cos 45° \sin\phi - 3\cos\phi - 6\sin\phi - R_B \sin 63.44 \cos\phi + R_B \cos 63.44 \sin\phi = 0$

On substituting:-

$\Sigma M_A = 0$

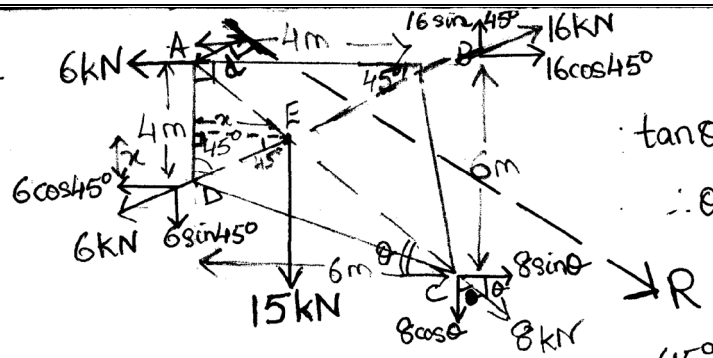
$-[R_B \cos(63.44+\phi)3] + (6\cos\phi \times 2) - (3\sin\phi \times 1) = 0$

$-3R_B \cos 63.44 \cos\phi - 3R_B \sin 63.44 \sin\phi + 12\cos\phi - 3\sin\phi = 0$

$9\cos\phi = 9\sin\phi \quad \therefore \tan\phi = 1 \quad \therefore \underline{\phi = 45°}$

Q3.19

(Q17)



$\tan\theta = \frac{6}{6} = 1$

$\therefore \theta = 45°$

$\Sigma F_x = 16\cos45° - 6 - 6\cos45° + 8\sin 45°$

$= 6.73$ kN $\rightarrow$

$\Sigma F_y = 16\sin45° - 6\sin45° - 15 - 8\cos 45°$

$= -13.585$ kN

$\therefore \Sigma F_y = 13.585 \downarrow$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$R = 15.16$ kN

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$

$\therefore \theta = 63.65°$

By Varignon's theorem, considering anticlockwise moments positive about A,

$\Sigma M_A^F = M_A^R$

$16\sin45° \times 4 - 6\cos45° \times 4 - 8\cos45° \times 6 + 8\sin45° \times 6 - 15 \times x = -R \times d$

For x: $BD = \sqrt{32} = 5.66$m ; $BE = \sqrt{8} = 2.83$m.

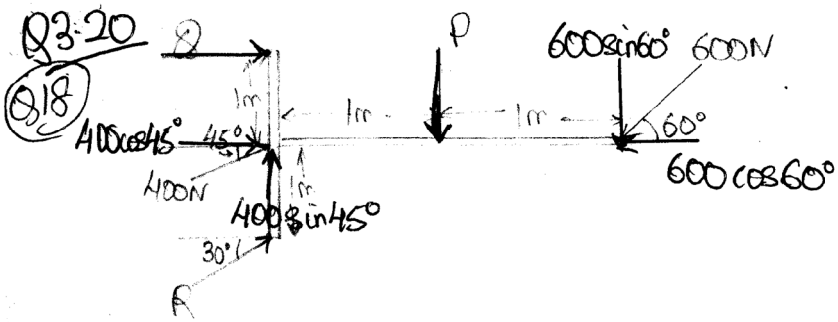
$DE = 2.83$m ; $\therefore x = 2$m.

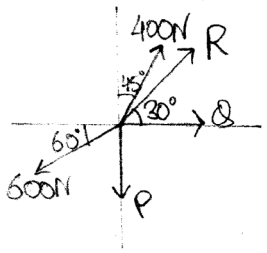
$\therefore -1.716 = -15.16 \times d$

$\therefore d = 0.113$m

∴ Additional force is $P = 15.16$kN at $\theta = 63.65°$ in IV Quadrant at a perpendicular distance of 0.113m to the right of A.

Q3.20 (Q18)





$\Sigma F_x = 0$

$Q + R\cos 30° + 400\cos 45° = 600\cos 60°$

$\therefore Q = 17.16 - 0.866R$ —— (I)

$\Sigma f_y = 0$

$R\sin 30° + 400\sin 45° = P + 600\sin 60°$

$\therefore P = 0.5R - 236.77$ —— (II)

$\Sigma M_Q = 0$

$-(P \times 1) - (600\sin 60° \times 2) - (Q \times 2) + (600\cos 60 \times 1) - (400\cos 45° \times 1) = 0$

$\therefore P + 2Q = -1022.1$ —— (III)

From (I), (II) & (III)

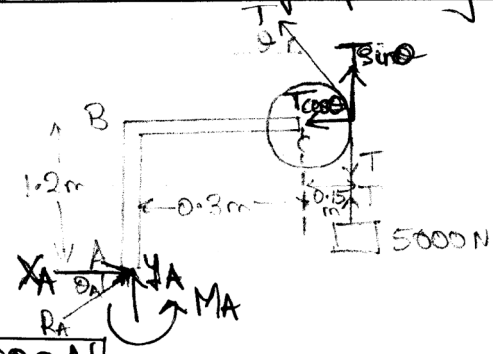
$R = 665.3N$ at $\theta = 30°$ in Ist quadrant

$P = 95.88\ N\ (\downarrow)$

$Q = -558.99N$

$\therefore Q = 558.99\ (\leftarrow)$

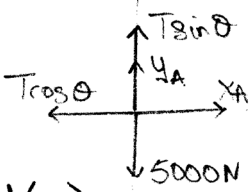
93.22 (819) Consider FBD of pulley and column



$\tan\theta = \frac{4}{3}$

$\therefore \theta = 53.13°$

$T = 5000\,N$



$\Sigma F_x = 0$

$\therefore X_A = T\cos\theta$

$\therefore X_A = 5000\cos 53.13 = 3000\,N(\rightarrow)$

$\Sigma F_y = 0$

$Y_A + T\sin\theta = 5000$

$Y_A = 5000 - 5000\sin 53.13 = 1000\,N(\uparrow)$

$\Sigma M_A = 0$

$+M_A = (5000 \times 0.45) + (T\cos\theta \times 1.29) + (T\sin\theta \times 0.42) = 0$

$M_A = -3300\,Nm$

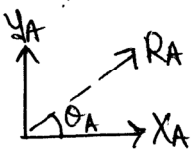
$\therefore M_A = 3300\,Nm$ (Clockwise)

$R_A = \sqrt{X_A^2 + Y_A^2}$

$R_A = 3162.28\,N$

$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right)$

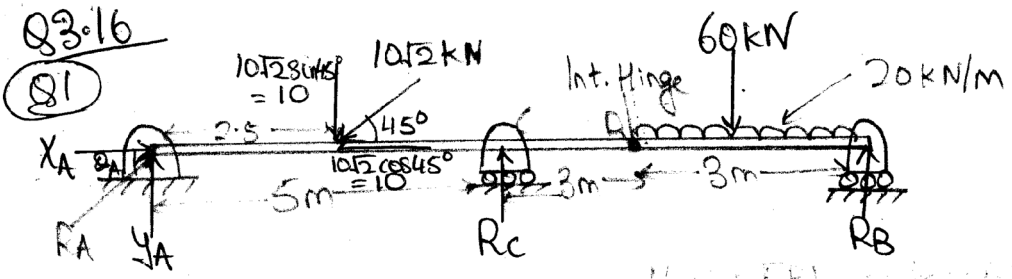
$\theta_A = 18.435°$



CE = 0.12
DE = 0.09
∴ BD = 0.42
AD = 1.29

Q3.16

Q1



$\Sigma F_x = 0$

$\therefore X_A = 10\,kN (\rightarrow)$

$\Sigma F_y = 0$

$Y_A - 10 + R_C - 60 + R_B = 0$

$\therefore Y_A + R_C + R_B = 70 \quad \ldots (I)$

$\Sigma M_A = 0$

$\therefore -(10 \times 2.5) + (R_C \times 5) - (60 \times 9.5) + (R_B \times 11) = 0$

$\therefore 5R_C + 11R_B = 595 \quad \ldots (II)$

$\Sigma M_B = 0$

$\therefore -(Y_A \times 11) + (10 \times 8.5) - (R_C \times 6) + (60 \times 1.5) = 0$

$\therefore +11Y_A + 6R_C = 175 \quad \ldots (III)$

From (I), (II) & (III)

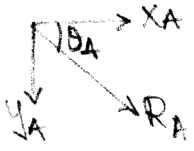
$Y_A = -13\,kN = 13\,kN \downarrow$

$R_C = 53\,kN (\uparrow)$

$R_B = 30\,kN (\uparrow)$

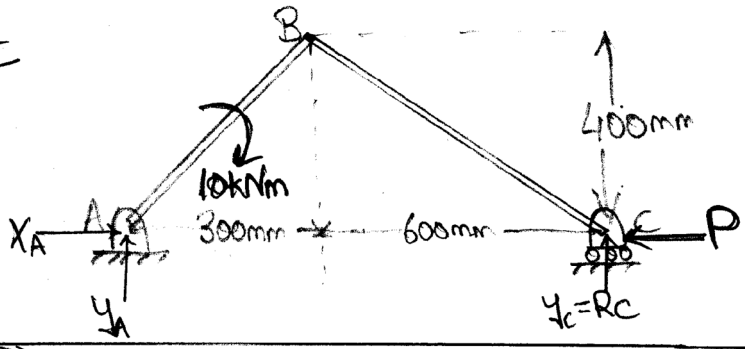
$R_A = \sqrt{X_A^2 + Y_A^2} = 16.4\,kN$

$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 52.43°$



Q3.17

Q2



$\Sigma F_x = 0$

$\therefore X_A = P \quad \ldots (I)$

$\Sigma F_y = 0$

$\therefore Y_A + Y_C = 0 \quad \ldots (II)$

$\Sigma M_C = 0$

$\therefore Y_A \times \frac{900}{1000} + 10 = 0$

$\therefore Y_A = -11.11\ kN$

$\therefore Y_C = 11.11\ kN$

$\Sigma M_B = 0$

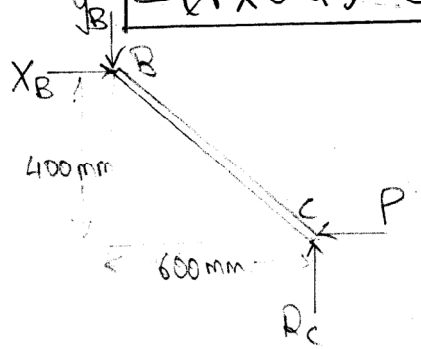
$\therefore (X_A \times 0.4) - (Y_A \times 0.3) - 10 + (Y_C \times 0.6) - (P \times 0.4) = 0$

Consider FBD of BC :-

$\Sigma M_B = 0$

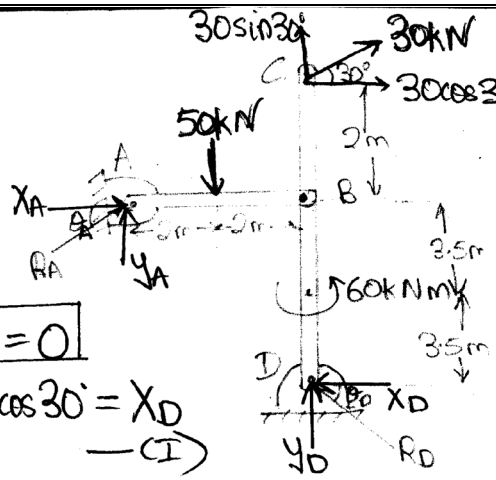
$\therefore R_C \times 0.6 = P \times 0.4$

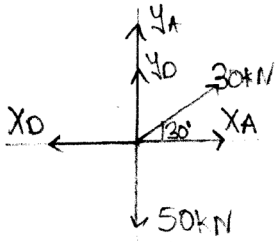
$\therefore P = 16.67\ kN (\leftarrow)$



Q3.21

Q3





$\Sigma F_x = 0$

$X_A + 30\cos 30^\circ = X_D$ — (I)

$\Sigma F_y = 0$

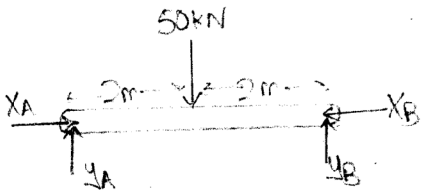
$Y_A + Y_D + 30\sin 30^\circ = 50$ — (II)

$\Sigma M_A = 0$

$-(50 \times 2) + (30\sin 30^\circ \times 4) - (30\cos 30^\circ \times 2) + (Y_D \times 4) - (X_D \times 7) + 60 = 0$

$4Y_D - 7X_D = 31.96$ — (III)

Consider FBD of AB:-



$\Sigma M_B = 0$

$(Y_A \times 4) - (50 \times 2) = 0$

$\therefore 4Y_A = 100$

$Y_A = 25\text{kN} (\uparrow)$

Subs. in (II)

$Y_D = 10\text{kN} (\uparrow)$

Subs. in (III)

$X_D = 1.15\text{kN} (\leftarrow)$

Subs. in (I)

$X_A = -24.83\text{kN}$

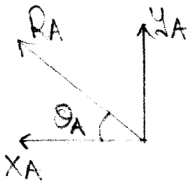
$\therefore X_A = 24.83\text{kN} (\leftarrow)$

$\therefore R_A = \sqrt{X_A^2 + Y_A^2}$

$R_A = 35.24\text{kN}$

$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right)$

$\theta_A = 45.2^\circ$

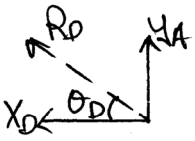


$\therefore R_B = \sqrt{X_B^2 + Y_B^2}$

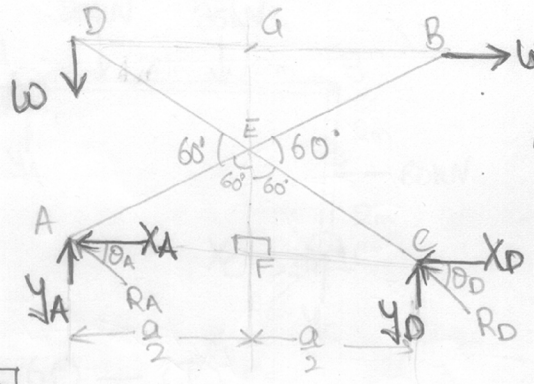
$R_B = 10.07\text{kN}$

$\theta_D = \tan^{-1}\left(\frac{Y_D}{X_D}\right)$

$\theta_D = 83.44^\circ$



Q 3.23 (04)



$EF = EG = \frac{a/2}{\tan 60°} = 0.29a$

$GF = 0.58a$

$\Sigma M_A = 0$

$(Y_D \times a) - (W \times 0.577a) = 0$

$Y_D = 0.577W \ (\uparrow)$

$\Sigma M_C = 0$

$-(W \times a) + (W \times 0.577a) + (Y_A \times a) = 0$

$\therefore Y_A = 0.423W \ (\uparrow)$

$\Sigma F_x = 0$

$X_A + X_D = W$

$\Sigma M_D = 0$

$-(X_A \times 0.577a) - (X_D \times 0.577a) + (Y_D \times a) = 0$

Consider FBD of AB:-

$\Sigma M_E = 0$

$-(W \times 0.289a) - (Y_A \times \frac{a}{2}) - (X_A \times 0.289a) = 0$

$\therefore X_A = -1.732W = 1.732W (\leftarrow)$

$X_D = 2.732W (\leftarrow)$

$R_D = \sqrt{X_D^2 + Y_D^2} = 2.792W$

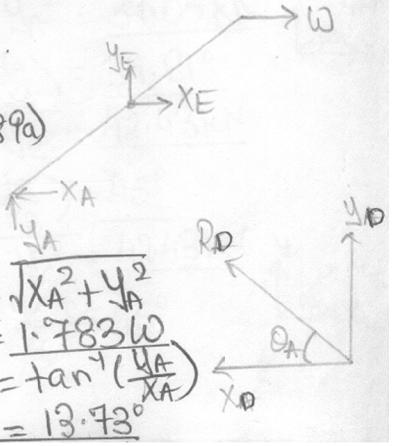
$\theta_D = \tan^{-1}\left(\frac{Y_D}{X_D}\right) = 11.93°$

$R_A = \sqrt{X_A^2 + Y_A^2}$

$R_A = 1.783W$

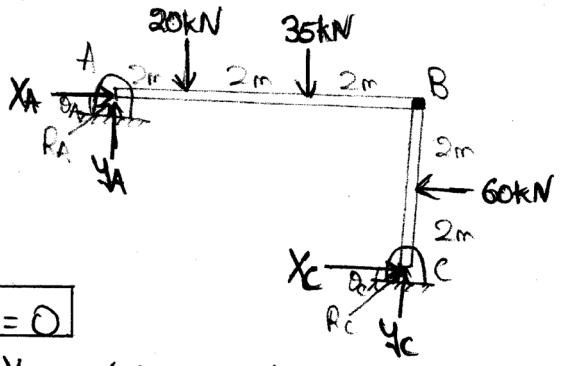
$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right)$

$\theta_A = 13.73°$



Q3.24

(05)



$\Sigma F_x = 0$

$X_A + X_C = 60$ — (I)

$\Sigma F_y = 0$

$Y_A + Y_C = 20 + 35$

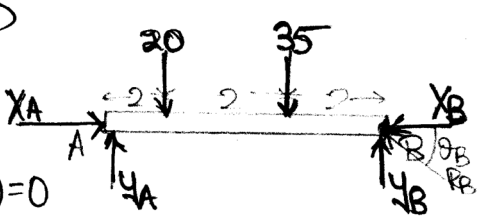
$Y_A + Y_C = 55$ — (II)

$\Sigma M_A = 0$

$(-20 \times 2) + (-35 \times 4) + (-60 \times 2) + (X_C \times 4) + (Y_C \times 6) = 0$

$4X_C + 6Y_C = 300$ — (III)

FBD of AB:-



$\Sigma M_A = 0$

$-(20 \times 2) + (-35 \times 4) + (Y_B \times 6) = 0$

$\therefore Y_B = 30\,kN$

$\Sigma M_B = 0$

$(Y_A \times 6) - (20 \times 4) - (35 \times 2) = 0$

$\therefore Y_A = 25\,kN\,(\uparrow)$

Subs. in (II)

$Y_C = 30\,kN\,(\uparrow)$

Subs. in (III)

$X_C = 30\,kN\,(\rightarrow)$

Subs. in (I)

$X_A = 30\,kN\,(\rightarrow)$

Subs. in (IV)

$X_B = 30\,kN$

$\Sigma F_x = 0$

$\therefore X_A = X_B$ — (IV)

$R_A = \sqrt{X_A^2 + Y_A^2} = 39.05\,kN$

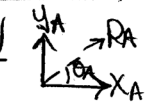
$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 39.81°$

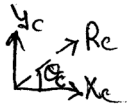
$R_B = \sqrt{X_B^2 + Y_B^2} = 42.43\,kN$

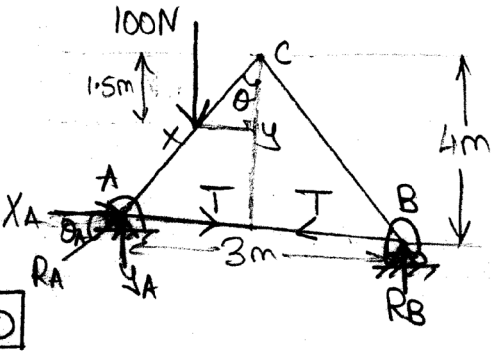
$\theta_B = \tan^{-1}\left(\frac{Y_B}{X_B}\right) = 45°$

$R_C = \sqrt{X_C^2 + Y_C^2} = 42.43\,kN$

$\theta_C = \tan^{-1}\left(\frac{Y_C}{X_C}\right) = 45°$







$\tan\theta = \frac{1.5}{4}$

$\therefore \theta = 20.56°$

$\tan\theta = \frac{xy}{Cy}$

$\therefore xy = 1.5 \tan 20.56$

$\therefore xy = 0.563\,m.$

$\Sigma F_x = 0$

$\therefore X_A = 0$

$\Sigma F_y = 0$

$y_A + R_B = 100 \quad \text{— (I)}$

$\Sigma M_A = 0$

$-[100 \times (\frac{1.5}{3} - 0.563)] + R_B \times 3 = 0$

$\therefore R_B = 31.23N\ (\uparrow)$

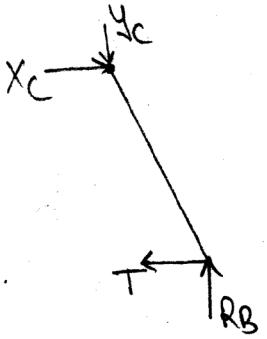
Subs. in (I)

$\therefore y_A = 68.77N\ (\uparrow)$

$R_A = \sqrt{X_A^2 + y_A^2} = 68.77N$

$\theta_A = 90°$

FBD of BC :-



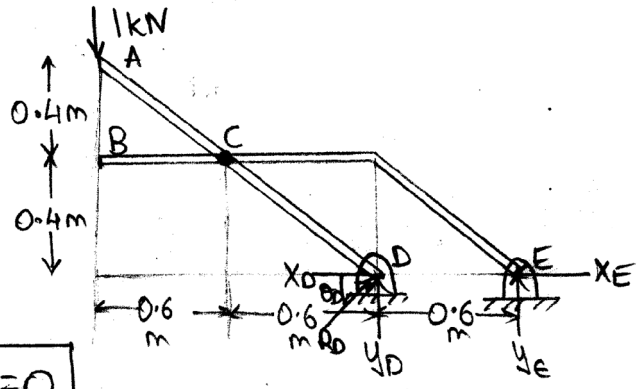
$\Sigma M_C = 0$

$(R_B \times 1.5) - (T \times 4) = 0$

$T = 11.718\,N$

Q3.26

Q7



$\Sigma F_x = 0$

$X_D = X_E$ — (I)

$\Sigma f_y = 0$

$y_E + y_D = 1$ — (II)

$\Sigma M_D = 0$

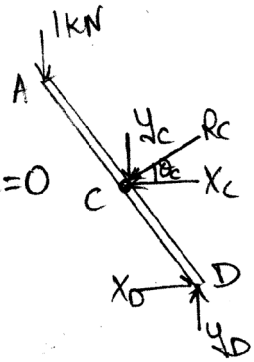
$(1 \times 1.2) + (y_E \times 0.6) = 0$

$\therefore y_E = -2\,kN = 2\,kN\,(\downarrow)$

Subs. in (II)

$\therefore y_D = +3\,kN\,(\uparrow)$

FBD of ACD



$\Sigma M_C = 0$

$(1 \times 0.6) + (y_D \times 0.6) + (X_D \times 0.4) = 0$

$X_D = -6\,kN = 6\,kN\,(\leftarrow)$

Subs. in (I)

$X_E = 6\,kN\,(\rightarrow)$

$\Sigma F_x = 0$

$X_C = X_D$

$X_C = 6\,kN$

$\Sigma f_y = 0$

$y_D = 1 + y_C$

$\therefore y_C = 2\,kN$

$R_C = \sqrt{X_C^2 + y_C^2} = 6.325\,kN$

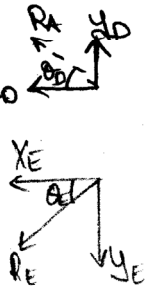
$\theta_C = \tan^{-1}\left(\frac{y_C}{X_C}\right) = 18.44^\circ$

$R_D = \sqrt{X_D^2 + y_D^2} = 6.71\,kN$

$\theta_D = \tan^{-1}\left(\frac{y_D}{X_D}\right) = 26.57^\circ$

$R_E = \sqrt{X_E^2 + y_E^2} = 6.325\,kN$

$\theta_E = \tan^{-1}\left(\frac{y_E}{X_E}\right) = 18.44^\circ$



FBD of sphere :-

$\Sigma F_y = 0$

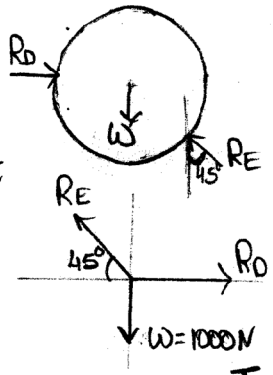
$R_E \sin 45° = W$

$\therefore R_E = 1414.2\,N$ at $\theta = 45°$ in II nd quadrant

$\Sigma F_x = 0$

$R_D = R_E \cos 45°$

$\therefore R_D = 1000\,N\ (\rightarrow)$

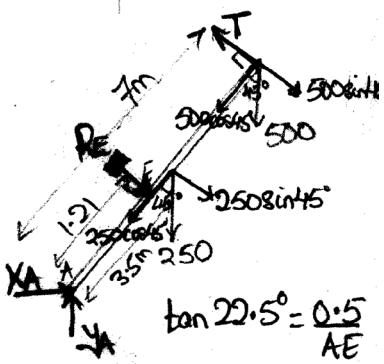


FBD of rod AB :-

$\Sigma M_A = 0$

$-(R_E \times 1.21) - (250\sin 45° \times 3.5) + (T \times 7) - (500\sin 45° \times 7) = 0$

$\therefore T = 686.4\,N$



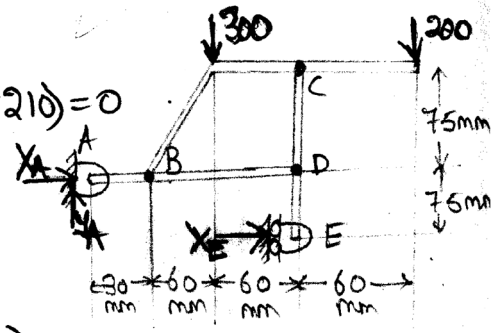
$\tan 22.5° = \frac{0.5}{AE}$

$\therefore AE = 1.21$

Q3.28 $\Sigma M_A = 0$

$(X_E \times 75) - (300 \times 90) - (200 \times 210) = 0$

$\therefore X_E = 920N$



$\Sigma F_x = 0$

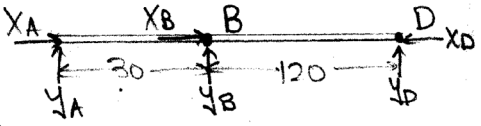
$X_A + X_E = 0$

$\therefore X_A = -920N = 920N (\leftarrow)$

$\Sigma F_y = 0$

$Y_A = 200 + 300 = 500N (\uparrow)$

FBD of ABD:-



$\Sigma M_B = 0$

$\therefore -(Y_A \times 30) + (Y_D \times 120) = 0$

$\therefore Y_D = 125N (\uparrow)$

$\Sigma F_y = 0$

$\therefore Y_A + Y_B + Y_D = 0$

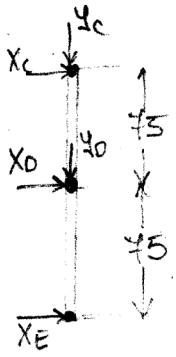
$\therefore Y_B = -625N = 625N (\downarrow)$

$\Sigma F_x = 0$

$X_A + X_B = X_D$

$\therefore X_B - X_D = 920 \quad — (I)$

FBD of CDE:-



$\Sigma M_C = 0$

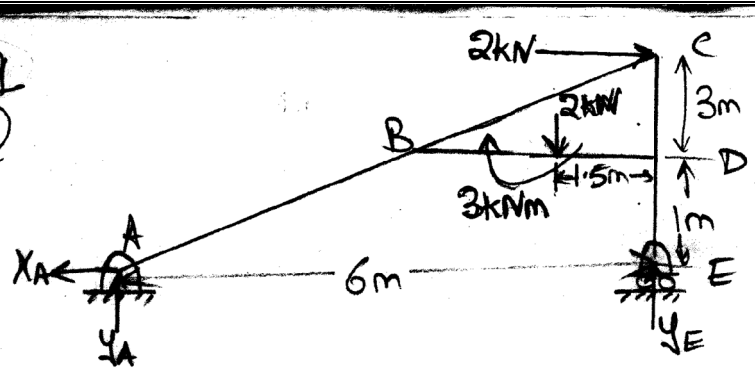
$+(X_D \times 75) + (X_E \times 150) = 0$

$X_D = -1840N = 1840N (\leftarrow)$

Subs. in (I)

$X_B = 920N = 920N (\leftarrow)$

3.29
(Q10)



$\Sigma F_x = 0$

$\therefore X_A = 2kN$

$\frac{3}{4} = \frac{BD}{6}$

$BD = 4.5m$

$\Sigma F_y = 0$

$\therefore Y_A + Y_E = 2 \quad — (I)$

$\Sigma M_A = 0$

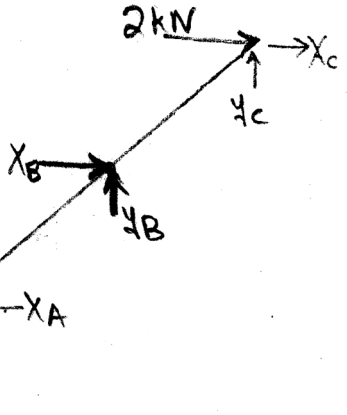
$-(2\times4) - (2\times4.5) - 3 + (Y_E \times 6) = 0$

$\therefore Y_E = 3.33kN$

Subs. in (I)

$\therefore Y_A = -1.33kN$

Consider FBD of ABC:-

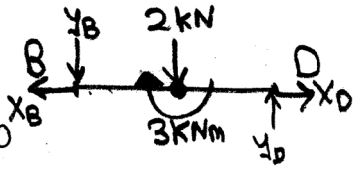


$\Sigma M_C = 0$

$X_B \times 3 - X_A \times 4 - Y_B \times 4.5 - Y_A \times 6 = 0$

$\therefore 3X_B - 4.5Y_B = 0 \quad (I)$

Consider FBD of BD:-

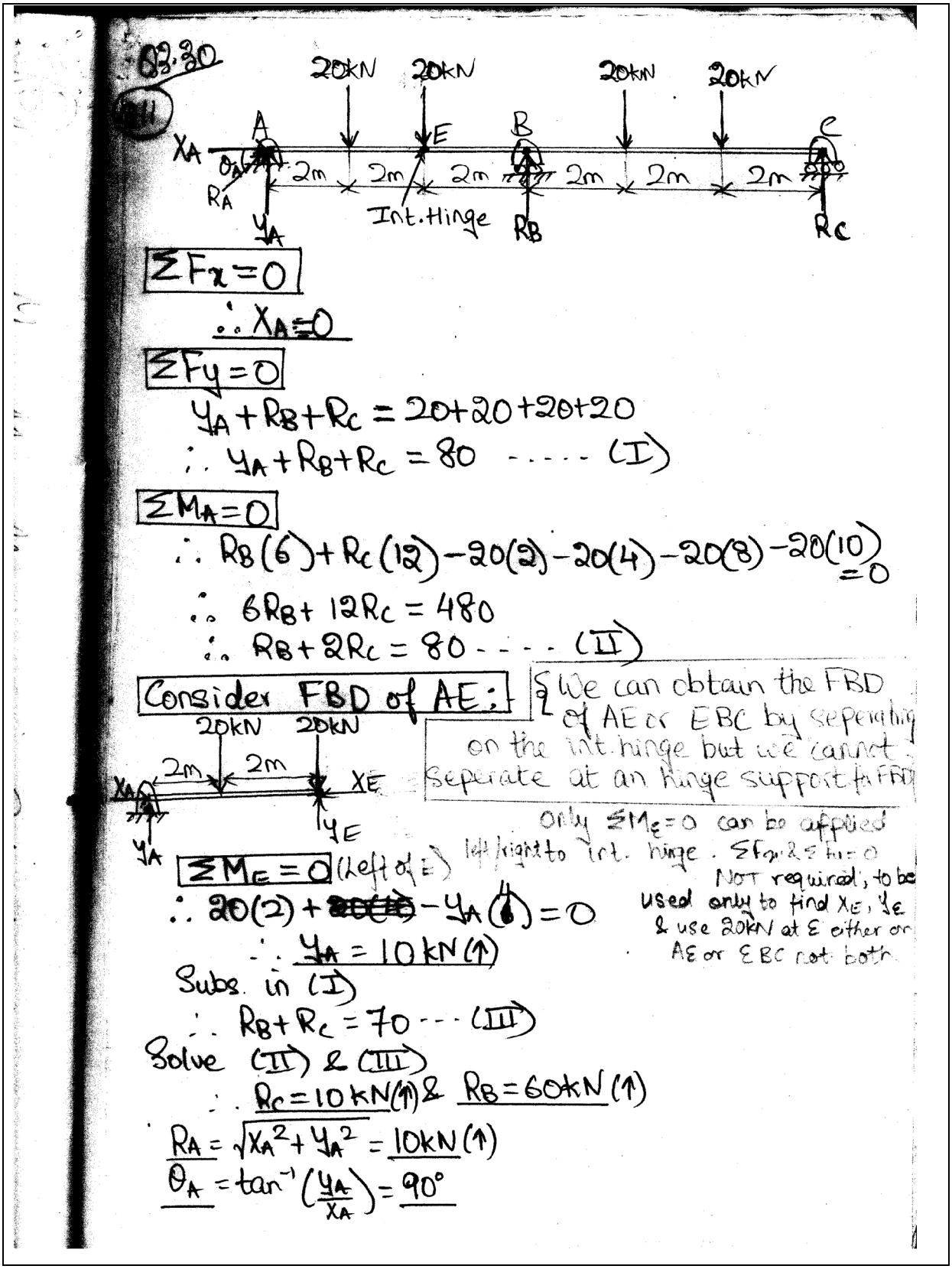


$\Sigma M_D = 0$

$-3 + Y_B \times 4.5 + 2 \times 1.5 = 0$

$\therefore Y_B = 0$

From (I) $X_B = 0$ $\therefore$ Force in pin B = 0.



$\Sigma F_x = 0$

$\therefore X_A = 0$

$\Sigma F_y = 0$

$Y_A + R_B + R_C = 20+20+20+20$

$\therefore Y_A + R_B + R_C = 80$ ..... (I)

$\Sigma M_A = 0$

$\therefore R_B(6) + R_C(12) - 20(2) - 20(4) - 20(8) - 20(10) = 0$

$\therefore 6R_B + 12R_C = 480$

$\therefore R_B + 2R_C = 80$ ..... (II)

Consider FBD of AE:

We can obtain the FBD of AE or EBC by seperating on the Int. hinge but we cannot seperate at an hinge support in FBD

$\Sigma M_E = 0$ (left of E)

Only $\Sigma M_E = 0$ can be applied left/right to Int. hinge. $\Sigma F_x = 0$ & $\Sigma F_y = 0$ NOT required, to be used only to find X_E, Y_E & use 20kN at E either on AE or EBC not both

$\therefore 20(2) + 20(4) - Y_A(6) = 0$

$\therefore Y_A = 10\,kN\,(\uparrow)$

Subs. in (I)

$\therefore R_B + R_C = 70$ ... (III)

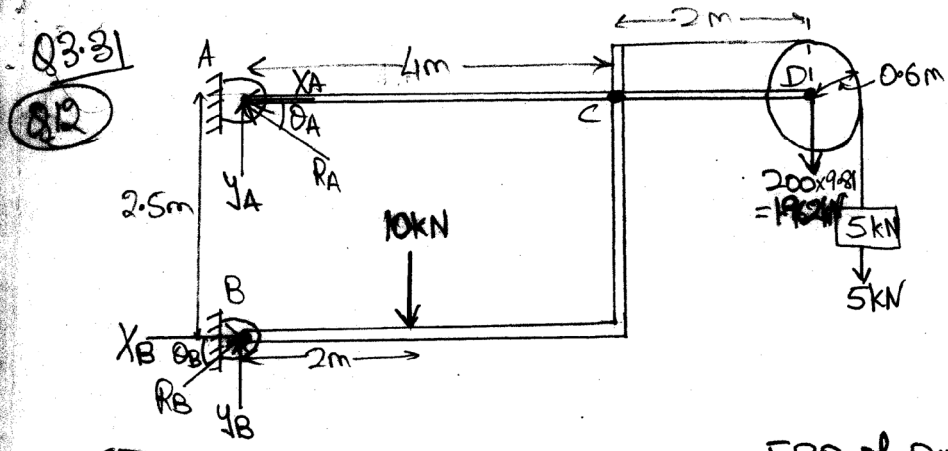
Solve (II) & (III)

$\therefore R_C = 10\,kN\,(\uparrow)$ & $R_B = 60\,kN\,(\uparrow)$

$R_A = \sqrt{X_A^2 + Y_A^2} = 10\,kN\,(\uparrow)$

$\theta_A = \tan^{-1}\left(\frac{Y_A}{X_A}\right) = 90°$

Q3.31



$\Sigma F_x = 0$

$\therefore X_A = X_B$ ---- (I)

$\Sigma F_y = 0$

$y_A + y_B = 1.962 + 5 + 10$

$y_A + y_B = 16.962$ ---- (II)

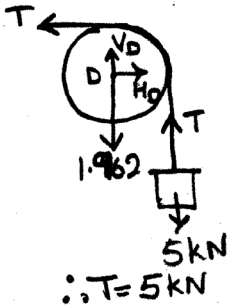
$\Sigma M_A = 0$

$X_B(2.5) - 1.962(6) - 5(6.6) = 10(2)$

$X_B = 25.91$ kN

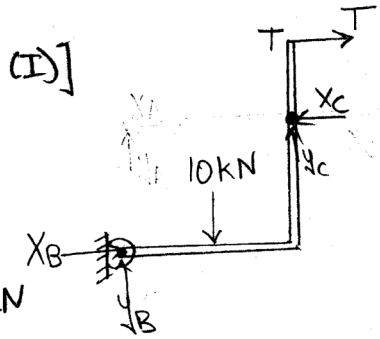
$X_A = 25.91$ kN [from (I)]

FBD of pulley



$\therefore T = 5$ kN

Consider FBD of BC:-



$\Sigma F_x = 0$

$\therefore X_B = X_C - T$

$\therefore X_C = 25.91$ kN FT

$\therefore X_C = 25.91 + 5 = 30.91$ kN

$\Sigma M_C = 0$

$\therefore (10 \times 2) + (X_B \times 2.5) - (y_B \times 4) - (T \times 0.6) = 0$

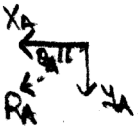
$\therefore y_B = 20.44$ kN

$\therefore y_A = -3.478$ kN [From (II)]

$\therefore y_A = 3.478$ kN ↓

$\Sigma F_y = 0$

$\therefore y_B + y_C = 10$ $\therefore y_C = 10.44$ kN

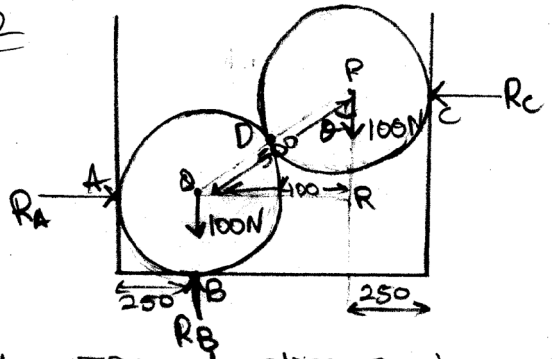


Reaction at A is $R_A = 26.14$ kN ; $\theta_A = 7.645°$

Reaction at B is $R_B = 33$ kN ; $\theta_B = 38.27°$

Pin reactions at C are $X_C = 30.91$ kN & $y_C = 10.44$ kN

Q3.32



PR = 300mm
$\sin\theta = \frac{400}{500}$ ∴ $\theta = 53.13°$

Consider FBD of entire system:- $\Sigma F_y = 0$

$R_B - 100 - 100 = 0$ ∴ $R_B = 200N (\uparrow)$

$\Sigma M_A = 0$

$\therefore (-100 \times 250) + (R_B \times 250) + (R_C \times 300) - (100 \times 650) = 0$

$\therefore R_B = R_C = 133.33N (\leftarrow)$

$\Sigma F_x = 0$

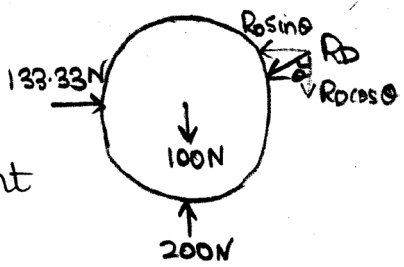
$\therefore R_A - R_C = 0$ ∴ $R_A = 133.33N (\rightarrow)$

Consider FBD of sphere Q:-

$\Sigma F_x = 0$

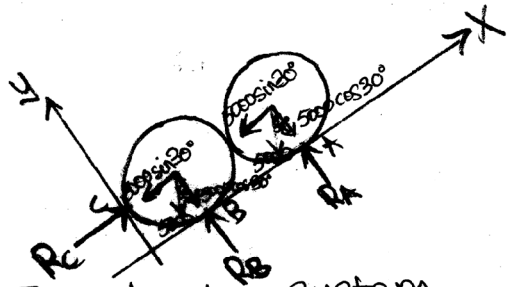
$\therefore 133.33 - R_Q \sin 53.13° = 0$

$\therefore R_Q = 166.66$ at $\theta = 53.13°$ in III rd quadrant



83·33

Q14



Consider FBD of entire system

$\Sigma F_x = 0$

$\therefore R_C - 5000 \sin 30° - 5000 \sin 30° = 0$

$\therefore R_C = 5000\ N$

$\Sigma F_y = 0$

$R_A + R_B = 5000 \cos 30° + 5000 \cos 30°$

$\therefore R_A + R_B = 8,660.25 \longrightarrow (I)$

$\Sigma M_C = 0$

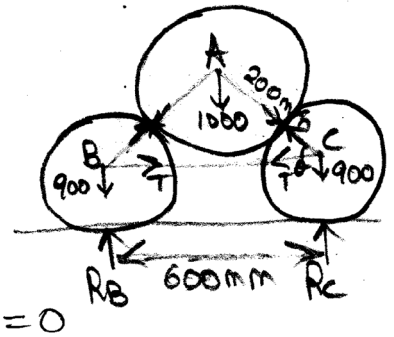
$\therefore (R_A \times 1.2) + (R_B \times 0.4) - (5000 \cos 30° \times 0.4) - (5000 \cos 30° \times 1.2) = 0$

$\therefore 1.2R_A + 0.4R_B = 6928.2 \longrightarrow (II)$

$\therefore R_A = 4330.125\ N$

$R_B = 4330.125\ N$

Applying conditions of equilibrium to entire system



$\Sigma F_y = 0$

$R_B + R_C - 1000 - 900 - 900 = 0$

$\therefore R_B + R_C = 2800 \quad \text{---(I)}$

$\Sigma M_B = 0$

$R_C \times 600 - 900 \times 600 - 1000 \times \frac{600}{2} = 0$

$\therefore R_C = 1400N$

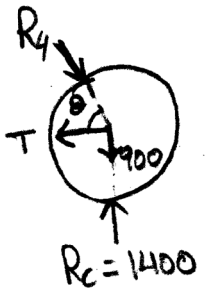
Subs. in (I) $\therefore R_B = 1400N$

$\cos\theta = \frac{300}{400}$

$\therefore \theta = 41.41°$

Consider F.B.D. of sphere C :-

Applying condition of equilibrium



$\Sigma F_y = 0$

$\therefore 1400 - 900 - R_4 \sin\theta = 0$

$\therefore R_4 = 755.92N$

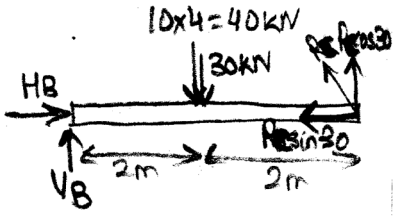
$\Sigma F_x = 0$

$-T + R_4 \cos\theta = 0$

$\therefore T = 566.94N$

16 Considering FBD of portion BC:-

Applying conditions of equilibrium

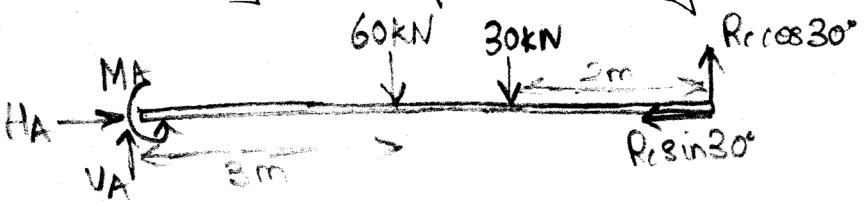


$\Sigma M_B = 0$

$R_c \cos 30° \times 4 - 30 \times 2 - 40 \times 2 = 0$

$\therefore R_c = 40.42\ kN$

Considering FBD of entire system:-



Applying conditions of equilibrium

$\Sigma F_x = 0$

$\therefore H_A - R_c \sin 30 = 0$

$\therefore H_A = 20.21\ kN\ (\rightarrow)$

$\Sigma F_y = 0$

$V_A + R_c \cos 30 - 60 - 30 = 0$

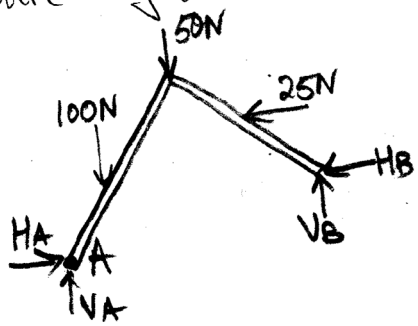
$V_A = 55\ kN\ (\uparrow)$

$\Sigma M_A = 0$

$\therefore -M_A + 60 \times 3 + 30 \times 4 - R_c \cos 30 \times 6 = 0$

$M_A = 89.97\ kNm\ (\circlearrowleft)$

∴ Considering FBD of entire system



$\Sigma F_x = 0$

$H_A - 25 - H_B = 0$

$\therefore H_A - H_B = 25 \quad \text{--- (I)}$

$\Sigma F_y = 0$

$V_A + V_B - 100 - 50 = 0$

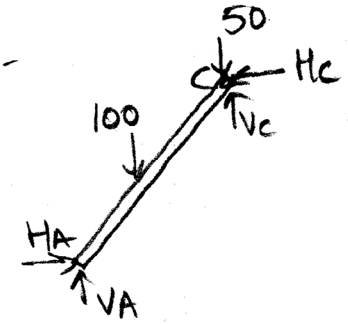
$\therefore V_A + V_B = 150 \quad \text{--- (II)}$

$\Sigma M_A = 0$

$-100 \times 1.5 - 50 \times 3 + 25 \times 3 + H_B \times 1 + V_B \times 7 = 0$

$\therefore H_B + 7V_B = 225 \quad \text{--- (III)}$

Considering FBD of rod AC:-



$\Sigma M_C = 0$

$\therefore H_A \times 4 - V_A \times 3 + 100 \times 1.5 = 0$

$\therefore 4H_A - 3V_A = -150 \quad \text{--- (IV)}$

From (I) $H_A = H_B + 25$

Subs. in (IV)

$\therefore 4H_B - 3V_A = -250 \quad \text{-- (V)}$

Solving eqns. (II), (III) & (IV) simultaneously

$\therefore H_B = 29\,N (\rightarrow)$

$V_A = 122\,N (\uparrow)$

$V_B = 28\,N (\uparrow)$

Subs. in (I)

$\therefore H_A = 54\,N (\rightarrow)$

18 Considering FBD of sphere A:-

Applying conditions of equilibrium.

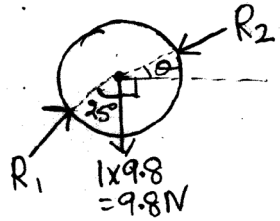
$\because \Sigma F_x = 0$

$\therefore R_1 \sin 25° - R_2 \cos\theta = 0 \quad \cdots (I)$

$\because \Sigma F_y = 0$

$\therefore R_1 \cos 25° - R_2 \sin\theta - 9.8 = 0$

$\therefore R_1 \cos 25° = 9.8 + R_2 \sin\theta \quad \cdots (II)$



Considering F.B.D of sphere B:-

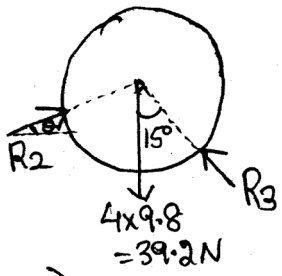
$\Sigma F_x = 0$

$\therefore R_2 \cos\theta - R_3 \sin 15 = 0$

$\therefore R_2 \cos\theta = R_3 \sin 15 \quad \cdots (III)$

$\Sigma F_y = 0$

$-39.2 + R_2 \sin\theta + R_3 \cos 15 = 0 \quad \cdots (IV)$



From eq (I); $\cos\theta = \frac{0.423R_1}{R_2}$

From eq (III); $\cos\theta = \frac{0.259R_3}{R_2}$

On equating we get $0.423R_1 = 0.259R_3 \quad \cdots (V)$

From eq (II); $\sin\theta = \frac{0.906R_1 - 9.8}{R_2}$

From eq (IV); $\sin\theta = \frac{0.966R_3 + 39.2}{R_2}$

On equating we get $0.906R_1 + 0.966R_3 = 49 \quad \cdots (VI)$

From (V) & (VI) $R_1 = 19.729N$

$R_3 = 32.221N$

$\because \cos^2\theta + \sin^2\theta = 1$

$\therefore \left(\frac{0.423R_1}{R_2}\right)^2 + \left(\frac{0.906R_1 - 9.8}{R_2}\right)^2 = 1 \quad \therefore R_2 = 11.612N$